Exact Metric Dimensions and Improved Upper Bounds for Hypercubes

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Abstract

Determining the metric dimension $\beta(Q_n)$ of the n-dimensional hypercube Q_n is a classical problem in graph theory and additive combinatorics with close connections to nonadaptive coin weighing and coin-detecting matrices. Despite extensive study over five decades, exact values were previously established only up to $n = 13$. In this paper, we resolve the next two values in the sequence, proving that

$$\beta(Q_{14}) = \beta(Q_{15}) = 9.$$

The lower bound is established via an exhaustive canonical-prefix search using Read–Faradžev orderly generation under the stabilizer of the normalized first row. The decisive computation prunes non-canonical prefixes through depth 8. Separate implementations in C and safe Rust agree on the full per-depth node-count vector, and the search is cross-checked against McKay's catalogue and reference enumerations of dissociated sets. As an immediate corollary, we settle the nonadaptive coin-weighing parameter $M(14) = 8$. Finally, using a meet-in-the-middle ternary kernel search combined with simulated annealing and extension scans, we construct explicit sign matrices, checked by two separately written exact verifiers, that improve the best known upper bounds for larger hypercubes:

$$\beta(Q_{24}) \le 13, \qquad \beta(Q_{26}) \le 14, \qquad \beta(Q_{29}) \le 15.$$

Keywords Metric dimension · Hypercube · Detecting matrix · Nonadaptive coin weighing · Orderly generation · Exhaustive search

Mathematics Subject Classification (2020) 05C12 · 05C85 · 90C27 · 11B13 · 68R05

1 Introduction

The vertices of the n-dimensional hypercube Q_n are the binary vectors $\{0,1\}^n$, with two vertices adjacent when they differ in exactly one coordinate. The graph distance $d(x, y)$ between vertices $x, y \in \{0,1\}^n$ is the standard Hamming distance. A subset of vertices $R = \{v_1, \dots, v_q\} \subseteq \{0,1\}^n$ is called a *resolving set* for Q_n if every vertex $x \in \{0,1\}^n$ is uniquely identified by its vector of distances to the vertices in R:

$$\big(d(x,v_1), d(x,v_2), \dots, d(x,v_q)\big) \neq \big(d(y,v_1), d(y,v_2), \dots, d(y,v_q)\big) \quad \text{for all } x \neq y.$$

A minimum resolving set is a *metric basis*. The *metric dimension* of Q_n, denoted by $\beta(Q_n)$, is the cardinality of a metric basis.

The metric-dimension problem was introduced independently by Slater [17] and by Harary and Melter [6]. For hypercubes, the problem has deep connections to Cartesian product dimensions, information theory, group testing, nonadaptive coin weighing, and dissociated vector sets in additive combinatorics [1, 2, 4, 9, 10, 16]. Although the asymptotic behavior $\beta(Q_n) = (2 + o(1))n/\log_2 n$ is known [4], computing exact small values is notoriously difficult due to the combinatorial explosion of the search space.

The OEIS sequence A303735 records the exact metric dimensions of hypercubes through $n = 13$ as

$$1, 2, 3, 4, 4, 5, 6, 6, 7, 7, 8, 8, 8$$

[14]. The first ten values were verified by V. S. Miller using MaxSAT solvers with symmetry-breaking constraints [14]. For dimensions $n = 14$ and $n = 15$, heuristic upper bounds $\beta(Q_{14}) \le 9$ and $\beta(Q_{15}) \le 9$ were discovered through genetic algorithms, variable neighborhood search, and integer programming heuristics [7, 8, 12, 13]. However, whether eight landmarks could suffice for $n = 14$ remained an open question.

In this paper, we resolve this question by proving that no 8-element resolving set exists for Q_{14}, establishing the following main theorem.

Theorem 1. *The following statements hold:*

(i) The exact metric dimensions of the 14- and 15-dimensional hypercubes are

$$\beta(Q_{14}) = \beta(Q_{15}) = 9.$$

(ii) The metric dimensions of the 24-, 26-, and 29-dimensional hypercubes satisfy the improved upper bounds

$$\beta(Q_{24}) \le 13, \qquad \beta(Q_{26}) \le 14, \qquad \beta(Q_{29}) \le 15.$$

Our nonexistence proof for 8×14 combines a normalized detecting-matrix formulation with an orderly generation algorithm that prunes non-canonical search-tree prefixes through depth 8. The accompanying repository provides the source code, final certificates, automated checks, reproduction harnesses, and summarized audit results [3]; the original raw discovery-campaign logs are not needed to check the theorem and are not distributed.

2 Detecting matrices and dissociated sets

Let $W \in \{-1, +1\}^{q \times n}$ be a sign matrix with rows $r_1, \dots, r_q \in \{-1, +1\}^n$. We call W a *detecting matrix* if its kernel contains no nonzero ternary vector:

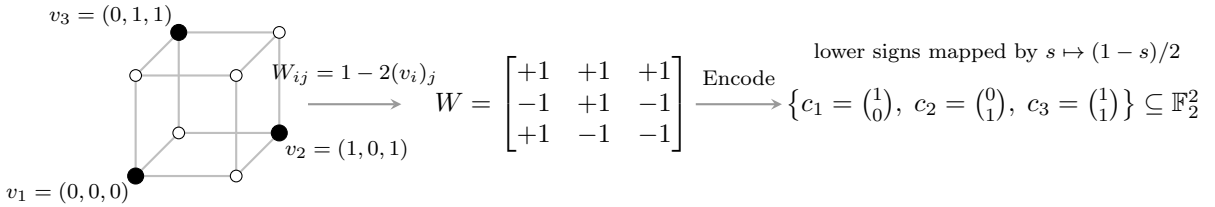
$$Wz \neq 0 \quad \text{for every nonzero } z \in \{-1, 0, 1\}^n.$$

Lemma 2 (Sebő–Tannier [16], Lu–Ye [10])**.** *A resolving set of size q in Q_n exists if and only if there exists a $q \times n$ detecting sign matrix $W \in \{-1, +1\}^{q \times n}$ satisfying $Wz \neq 0$ for all nonzero $z \in \{-1, 0, 1\}^n$.*

Proof. Let $v_1, \dots, v_q \in \{0, 1\}^n$ be candidate landmarks. Setting $W_{ij} = 1 - 2(v_i)_j$ maps $0 \mapsto +1$ and $1 \mapsto -1$. For any $x, y \in \{0, 1\}^n$, the distance difference to landmark v_i is

$$d(x, v_i) - d(y, v_i) = \sum_{j=1}^{n} (x_j - y_j)(1 - 2(v_i)_j) = \sum_{j=1}^{n} z_j W_{ij},$$

where $z = x - y \in \{-1, 0, 1\}^n$. Vertices $x \neq y$ have identical metric coordinates if and only if $Wz = 0$. □



(a) Hypercube Q_3 Landmarks $R = \{v_1, v_2, v_3\}$ | **(b) Sign Matrix** $Wz \neq 0\ \forall z \neq 0$ | **(c) Binary Column Types in $\mathbb{F}_2^2$ Distinct Sums at Each Subset Size**

Figure 1: The geometric and algebraic framework connecting hypercube metric bases, detecting sign matrices, and normalized column types in $\mathbb{F}_2^{q-1}$. Binary subset sums are taken in $\mathbb{Z}^{q-1}$ and must be distinct among subsets of equal cardinality; the omitted all-ones row distinguishes different cardinalities.

Proposition 3 (Cantor–Mills [2], Lindström [9])**.** *A sign matrix $W \in \{-1,+1\}^{q\times n}$ is detecting if and only if all 2^n subset sums of its column vectors are pairwise distinct in $\mathbb{Z}^q$ (i.e., its columns form a dissociated set).*

2.1 Column normalization

Multiplying any column of W by -1 preserves the detecting property: if D is a diagonal sign matrix, then $WDz = 0 \iff W(Dz) = 0$, and D is an automorphism of $\{-1,0,1\}^n$. We may therefore multiply each column by its first coordinate, normalizing the first row of W to all $+1$:

$$W = \begin{bmatrix} +1 & +1 & \cdots & +1 \\ & & W' & \end{bmatrix}.$$

Under this normalization:

(i) The first coordinate of the subset sum for any subset $A \subseteq [n]$ is simply the cardinality $|A|$. Consequently, subsets of different cardinalities can never collide.

(ii) Each column is fully determined by its lower $q-1$ coordinates. Under the bijection $s \mapsto \frac{1-s}{2}$, these coordinates map to binary vectors in $\mathbb{F}_2^{q-1}$.

(iii) No two columns can be identical, as $c_i = c_j$ implies $c_i - c_j = 0$.

Therefore, searching for a $q \times n$ detecting matrix is equivalent to searching for an n-element subset of the 2^{q-1} available *column types* $V = \{w \in \{-1,+1\}^q : w_1 = +1\} \cong \mathbb{F}_2^{q-1}$ whose subset sums in $\mathbb{Z}^q$ are all distinct. Figure 1 summarizes this correspondence.

3 Incremental search and exact packed arithmetic

Let $P = (c_1 < c_2 < \cdots < c_k)$ be an ordered sequence of column types chosen from V. Let $\Sigma(P) = \{\sum_{c\in A} c : A \subseteq P\} \subset \mathbb{Z}^q$ denote the set of all 2^k subset sums of P.

If all sums in $\Sigma(P)$ are distinct, adjoining a new column candidate $c \in V$ is collision-free if and only if

$$\Sigma(P) \cap \big(\Sigma(P) + c\big) = \varnothing. \tag{1}$$

When (1) holds, the new subset sum collection is the disjoint union $\Sigma(P \cup \{c\}) = \Sigma(P) \cup (\Sigma(P) + c)$.

The tree traversal exploits two monotone pruning rules:

(a) *Monotone candidate filtering:* If candidate c violates (1) for prefix P, it remains invalid for any extension $P' \supset P$, because the witnessing subset sums persist.

(b) *Lookahead capacity pruning:* If fewer than $n-k$ valid candidate columns remain in the filtered candidate list, prefix P cannot reach n columns and is pruned immediately.

3.1 Exact 64-bit packed coordinate arithmetic

For the critical 8×14 search, we pack all 8 coordinates of a subset sum into a single 64-bit unsigned machine integer (`uint64_t`). Each coordinate occupies one 8-bit byte. Define

$$P(v) = \sum_{j=0}^{q-1} (v_j + 64) 2^{8j}, \qquad \Delta(c) = \sum_{j=0}^{q-1} c_j 2^{8j}.$$

With at most 14 columns, each coordinate v_j satisfies $-14 \le v_j \le 14$, so biased values lie strictly in $[50, 78] \subset [0, 255]$. The code computes $P(v) + \Delta(c) = P(v + c)$: the bias is applied once, and the lane values remain far enough from a byte boundary that no carry or borrow crosses between lanes. Thus the packed `uint64_t` operation exactly implements simultaneous 8-dimensional integer-vector addition.

4 Symmetry reduction and orderly generation

The full automorphism group preserving detecting matrices consists of row signed permutations $O(q, \mathbb{Z}) = (\mathbb{Z}_2)^q \rtimes S_q$ (of order $2^q q!$), combined with column permutations S_n and column sign flips $(\mathbb{Z}_2)^n$. To eliminate redundant isomorphic subtrees, we integrate Read–Faradžev orderly generation [5, 15] directly into the tree search.

4.1 The normalized group action

For any signed row isometry $g \in O(q, \mathbb{Z})$ acting on $\{-1, +1\}^q$, we define its normalized action $\Phi_g : V \to V$ by

$$\Phi_g(w) = (g(w))_1 \cdot g(w).$$

Because $(g(w))_1 \in \{-1, +1\}$, the first coordinate of $\Phi_g(w)$ is always $+1$, mapping V to V.

Lemma 4. *For any $g \in O(q, \mathbb{Z})$, the map $\Phi_g : V \to V$ is a bijection on the type set V.*

Proof. Suppose $\Phi_g(u) = \Phi_g(v)$. Then $g(u) = \sigma g(v) = g(\sigma v)$ where $\sigma = (g(u))_1 (g(v))_1 \in \{-1, +1\}$. By invertibility of g, $u = \sigma v$. Comparing first coordinates gives $u_1 = +1 = \sigma v_1 = \sigma$, whence $\sigma = +1$ and $u = v$. Since V is finite, Φ_g is a bijection. □

The normalized group action decomposes into:

(a) *Stabilizer of the first row:* The subgroup $\mathrm{Stab}(\text{row } 1) \cong (\mathbb{Z}_2)^{q-1} \rtimes S_{q-1}$, of order $2^{q-1}(q-1)!$, acts on the type vectors $x \in \mathbb{F}_2^{q-1}$ as affine transformations:

$$x \longmapsto \pi(x \oplus t),$$

where $t \in \mathbb{F}_2^{q-1}$ denotes row sign flips on the lower $q-1$ rows, and $\pi \in S_{q-1}$ permutes the coordinates.

(b) *Row pivots:* Group elements swapping a different row $R \in \{2, \ldots, q\}$ into the first row act linearly on type bits $b = (b_0, \ldots, b_{q-2}) \in \mathbb{F}_2^{q-1}$ as

$$b'_0 = b_k, \qquad b'_j = b_k \oplus b_{\sigma(j)} \quad \text{for } j \geq 1,$$

where $k = R - 2$ and σ permutes the remaining coordinates.

4.2 Canonical-prefix pruning

We order the 2^{q-1} column types by integer value $0 < 1 < \cdots < 2^{q-1} - 1$. Every k-subset of types is represented as a sorted sequence $P = (c_1 < c_2 < \cdots < c_k)$.

Under translation $x \mapsto x \oplus c_1$, the first element is mapped to 0. Since 0 is the minimal type, every solution orbit contains a representative starting with 0. We therefore fix $c_1 = 0$ at depth 1.

Proposition 5 (Canonical-prefix rule)**.** *Let $X = (x_1 < \cdots < x_n) \subset V$ be the lexicographically minimal representative of its orbit under the normalized group action. Then for every $k \in [n]$, the initial prefix $A = (x_1 < \cdots < x_k)$ is lexicographically minimal among all sorted images $\Phi_g(A)$ that contain zero.*

Proof. Let A be the first k elements of X. Since Φ_g is a bijection, $\Phi_g(A) \subseteq \Phi_g(X)$. Let the sorted images be $\Phi_g(A) = (y_1 < \cdots < y_k)$ and $\Phi_g(X) = (z_1 < \cdots < z_n)$.

Because $\Phi_g(A)$ is a k-subset of $\Phi_g(X)$, the j-th smallest element of the superset cannot exceed the j-th smallest element of the subset:

$$z_j \leq y_j \qquad \text{for all } 1 \leq j \leq k.$$

Suppose for contradiction that some $g \in O(q, \mathbb{Z})$ produces $\Phi_g(A)$ containing 0 with $\Phi_g(A) <_{\text{lex}} A$. Let $m \in [k]$ be the first index where $y_m < x_m$ (with $y_j = x_j$ for $j < m$). Then for all $j < m$, $z_j \leq y_j = x_j$.

- If $z_j < x_j$ for any $j < m$, then $\Phi_g(X) <_{\text{lex}} X$.
- If $z_j = x_j$ for all $j < m$, then at index m we have $z_m \leq y_m < x_m$, which again yields $\Phi_g(X) <_{\text{lex}} X$.

In either case, $\Phi_g(X)$ is strictly lexicographically smaller than X, contradicting the minimality of X. □

Figure 2 illustrates the resulting canonical-prefix search tree, in which dashed branches are eliminated by collision testing or by prefix minimality.

4.3 Popcount filtering and pruning efficiency

Testing $(q-1)! = 5{,}040$ coordinate permutations at every node is accelerated by a popcount (Hamming weight) lower bound. Coordinate permutations preserve the Hamming weight of each column type. For a type with popcount w, the minimum integer value under any permutation is $\min_w = 2^w - 1$. Comparing these precomputed bounds against the incumbent prefix often settles the ordering in $O(1)$ time, invoking the full permutation check only when the bound is inconclusive.

Table 1 illustrates the pruning efficiency under the normalized-first-row stabilizer used by the decisive computation.

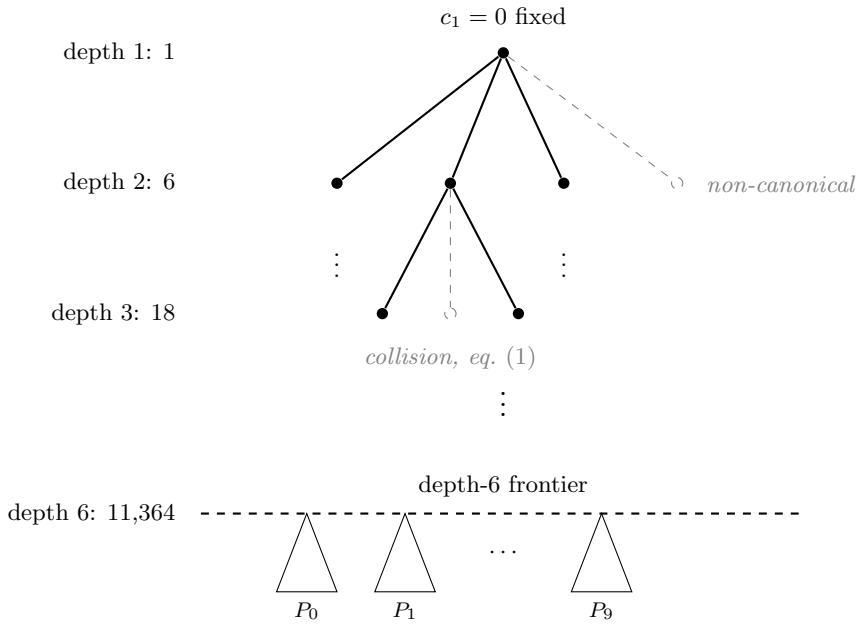


Figure 2: Schematic of the canonical-prefix search tree for 8×14. Each node represents a sorted, collision-free prefix with $c_1 = 0$. Dashed branches are pruned by collision testing (1) or canonical-prefix minimality (Proposition 5). The depth-6 frontier of 11,364 prefixes is partitioned into 10 disjoint subtrees $P_0, \ldots, P_9$, each verified to contain zero solutions.

Table 1: Effect of canonical-prefix pruning on search tree size.

Instance	No lex-min pruning	First-row stabilizer
7×11, total search nodes	256,123,828	189,831
8×14, nodes at depth 5	1,282,411	1,082
8×14, nodes at depth 6	34,942,921	11,364
8×14, complete search nodes	—	339,979,093

5 Search tree partitioning and node accounting

The 8×14 computation is partitioned across 10 deterministic processes at depth 6. Each process traverses the identical top tree down to depth 6. At the depth-6 frontier, each outgoing branch is assigned to partition $p \in \{0, \ldots, 9\}$ if its sequence counter satisfies counter $\equiv p \pmod{10}$.

Every partition outputs the identical shallow frontier vector:

$$(N_0, N_1, N_2, N_3, N_4, N_5, N_6) = (0, 1, 6, 18, 143, 1082, 11364).$$

The sum of visits across all ten partitions is 340,092,619. Since the 12,614 shallow nodes ($\sum_{i=0}^{6} N_i = 12{,}614$) are visited by all 10 processes, subtracting the 9 duplicate copies yields the exact distinct node total:

$$340{,}092{,}619 - 9 \times 12{,}614 = \mathbf{339{,}979{,}093}.$$

Each partition terminates with an explicit exit status confirming zero solutions.

Table 2: Computational verification and consistency checks.

Check	Scope and Outcome
Separate implementations	Standalone C and safe Rust implementations match the full per-depth node-count vector exactly.
McKay 12-set catalogue	All 118,485 McKay dissociated 12-set orbits in $\{0,1\}^7$ occur in the 8×13 output with zero missing [11].
Small-instance agreement	Exact raw solution sets match exhaustive reference enumerations for 3×4, 4×4, and 5×6.
Orbit canonicalization	All 13 separately generated canonical 5×6 orbit representatives match the reference generator.
Known OEIS ladder	Reproduces exact known sequence values $\beta(Q_{11}) = \beta(Q_{12}) = \beta(Q_{13}) = 8$ [14].
Memory & runtime safety	The small-instance reference suite for `enum.c` passes under AddressSanitizer and UndefinedBehaviorSanitizer.
Audit verification	A separate fail-closed checker validates log structure, hashes, partition totals, and zero-solution exit status.

Table 3: New upper bounds certified by explicit detecting matrices.

Dimension n	Matrix Rows q	Published Upper Bound	New Upper Bound $\beta(Q_n)$
24	13	14 [7]	**13**
26	14	15 [7]	**14**
29	15	16 [10]	**15**

6 Computational verification and consistency checks

To increase confidence in the computation, we applied the controls summarized in Table 2. These checks were developed within the same project and rely on the same orderly-generation argument; they are neither a third-party replication nor a formally verified proof certificate.

Proof of Theorem 1(i). The exhaustive computation proves that no 8×14 detecting matrix exists. By Lemma 2, $\beta(Q_{14}) \geq 9$. The explicit 9×15 detecting matrix exhibited in Appendix A yields, by deleting any one column, a 9×14 detecting matrix, establishing $\beta(Q_{14}) \leq 9$. Thus $\beta(Q_{14}) = 9$.

If an 8×15 detecting matrix existed, deleting any column would yield an 8×14 detecting matrix. Hence $\beta(Q_{15}) \geq 9$. The same 9×15 matrix provides the matching upper bound, giving $\beta(Q_{15}) = 9$. □

7 New upper bounds for large hypercubes

The upper bounds in Theorem 1(ii) are certified by three explicit sign matrices given in Appendix A. Table 3 compares them with the previously published bounds.

7.1 Search methods: meet-in-the-middle ternary kernel counter

The matrices were discovered using a meet-in-the-middle ternary kernel counter. Splitting n columns into two halves of size $\lceil n/2 \rceil$ and $\lfloor n/2 \rfloor$, the routine enumerates the signed sums of both halves,

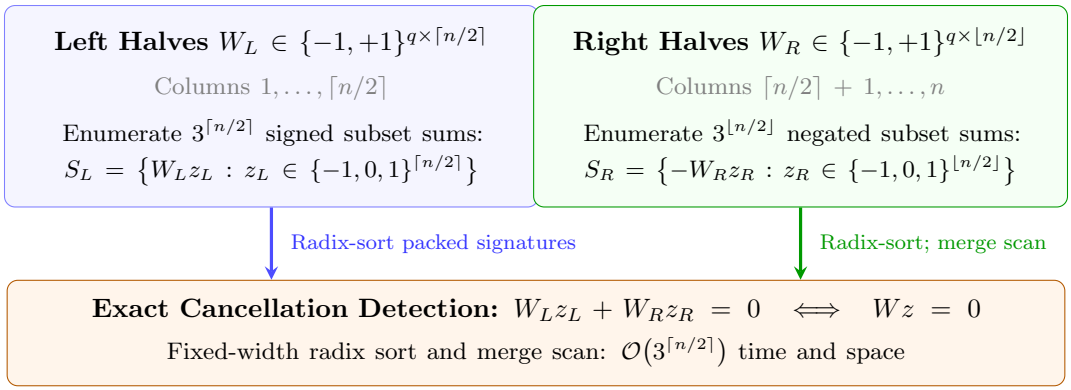


Figure 3: Meet-in-the-middle ternary kernel counter using fixed-width radix sorting and a merge scan for cancelling signatures.

fixed-width radix-sorts their packed row signatures, and merge-scans the sorted lists for exact cancellations $Wz = 0$. For fixed packed width, this requires $O(3^{\lceil n/2\rceil})$ time and space. For 15×29, the larger half has $3^{15} = 14{,}348{,}907$ signed choices. Figure 3 outlines the procedure.

The 15×29 certificate was found via bounded-weight simulated annealing. The 14×26 certificate was extracted as a submatrix slice, and the 13×24 certificate was constructed via an extension scan that tested all 2^{13} possible column extensions against a solved 13×23 matrix.

Proof of Theorem 1(ii). For each matrix in Appendix A, two separately written exact meet-in-the-middle verifiers confirm that no nonzero $z \in \{-1, 0, 1\}^n$ satisfies $Wz = 0$. By Lemma 2, this certifies the stated upper bounds. □

8 Nonadaptive coin weighing

In nonadaptive coin weighing, $M(n)$ denotes the minimum number of spring-scale weighings required to identify all subsets of n coins with known equal genuine weight. The classic coin-weighing problem originated with Erdős and Rényi [4], Cantor and Mills [2], and Lindström [9]. Its algebraic connection with hypercube metric dimension and minimal doubly resolving sets was developed by Sebő and Tannier [16] and Lu and Ye [10]:

$$\beta(Q_n) - 1 \leq M(n) \leq \beta(Q_n).$$

Corollary 6. *The exact nonadaptive coin-weighing value for 14 coins is* $M(14) = 8$.

Proof. Theorem 1(i) gives $M(14) \geq \beta(Q_{14}) - 1 = 8$. The accompanying artifact [3] exhibits an 8×14 binary matrix whose $2^{14} = 16{,}384$ subset sums are all distinct, confirming $M(14) \leq 8$. □

9 Reproducibility and future directions

The public repository contains the source code, final certificates, automated checks, reproduction harnesses, and summarized audit results [3]. The automated suite verifies all four sign-matrix certificates and the separate weighing certificate, exercises negative controls, and cross-validates small cases against direct reference enumerations. A separate reproduction harness checks the 8×13 classification against McKay's catalogue. Reproduction scripts can create non-overwriting raw log

bundles; the raw discovery campaign is not needed to check the theorem and is not distributed. AI-assisted development and writing are disclosed below; the author reviewed and remains responsible for every mathematical and computational claim.

For the next open parameter, $\beta(Q_{16})$, the repository provides a dynamic frontier framework that pairs full row pivoting with a local pool of worker processes on a multicore machine. Establishing $\beta(Q_{16})$ and producing machine-checkable SAT/DRAT certificates remains the principal open frontier.

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A Detecting-matrix certificates

Each block below displays an exact detecting sign matrix, where `+` denotes $+1$ and `-` denotes -1. By Lemma 2, the absence of a nonzero ternary kernel vector certifies the metric dimension upper bound.

A.1 A 9×15 certificate

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A.2 A 13×24 certificate

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A.3 A 14×26 certificate

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A.4 A 15×29 certificate

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Statements and Declarations

Funding. No funding was received for conducting this study.

Competing interests. The author declares no competing interests.

Data availability. All data supporting the findings of this study, including the detecting-matrix certificates and the per-depth node-count vectors, are contained in this article and in the public repository [3].

Code availability. The complete source code for the search and verification programs, together with the automated verification suite and reproduction harnesses, is publicly available at [3].

Declaration of generative AI and AI-assisted technologies. During this project, the author used Anthropic Claude, OpenAI Codex, and Antigravity to assist with software development, computational experimentation and verification, and manuscript drafting and revision. The author reviewed and validated the mathematical arguments, source code, computational outputs, citations, and final text, and takes full responsibility for the work.

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